PLASMON NEUTRINOS EMISSION IN A STRONG MAGNETIC FIELD

I: *Transverse Plasmons*

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Abstract. In this paper we generalize the Adams, Ruderman, Woo and Zaidi plasmon decay process to include the presence of a strong magnetic field. Two cases are studied; propagation parallel and perpendicular to the magnetic field. In either case we found that relevant changes only show for $H \simeq 10^{12}$–10^{13} G.

1. Introduction

In this paper we shall continue the investigation on the effects of an intense magnetic field on the rate of the elementary processes in stellar interiors (Canuto and Chiu, 1969; Fassio-Canuto, 1969).

It has been shown by Adams, Ruderman, Woo (1963)** and subsequently by Zaidi (1965) that due to the presence of plasma, a photon, acquiring a finite mass, $m_\gamma \equiv \hbar\omega p/c^2$, can decay into a pair of neutrinos and anti-neutrinos which, as is well known, escape without further interaction from the star. Since the above analysis has been applied to objects like white dwarfs, where it is likely to find an intense magnetic field, it appears interesting to study the effect of the field on such a process.

In this paper we present a computation of the energy loss in a star due to the decay of a transverse plasma in a neutrino-antineutrino pair.

2. The Lagrangian of the Process

The basic Feynman diagram for plasmon decay is shown in Figure 1. At the point x we have an electromagnetic interaction described as usually by

$$L(x) = ie\bar{\psi}(x)\,\gamma_\mu\psi(x)\,A_\mu(x)\,, \tag{1}$$

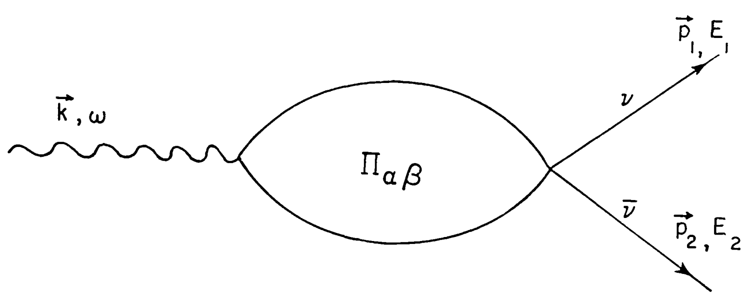

Fig. 1. Feynman diagram for plasmon decay into a neutrino-anti-neutrino pair.

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** Hereafter referred to as ARW.

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where $\psi(x)$ is the wave function of the electron in a magnetic field and A_μ the plasmon plane wave. At the point y we have a weak interaction Lagrangian given by

$$L(y) = \frac{g}{\sqrt{2}} (\bar{\psi}\gamma_\mu\psi)(\bar{\phi}\gamma_\mu(1+\gamma_5)\phi), \tag{2}$$

where g is the weak interaction coupling constant and ϕ describes the neutrino field. The S-matrix of the process is given by

$$S = \int\int \mathrm{d}x^4 \, \mathrm{d}y^4 T[L(x) \, L(y)], \tag{3}$$

where T is the usual ordering operator. Standard methods of QED are used to compute the energy loss

$$Q = \iint \frac{\overline{|S|^2}}{\Omega T} (E_1 + E_2) \frac{\Omega^2}{(2\pi\hbar)^6} \, \mathrm{d}^3 p_1 \, \mathrm{d}^3 p_2 \, . \tag{4}$$

These methods will be briefly discussed in Section 4.

3. The Plasma Parameters

The potential A_μ describes the electromagnetic field of a transverse plasma wave and can be written in the following form

$$A_\mu(x) = \sum_{\mathbf{k}} \sum_{l=1}^{2} N_\gamma^{1/2} \{a_l(\mathbf{k}) \, e^{ikx} + a_l^\dagger(\mathbf{k}) \, e^{-ikx}\} \, e_\mu^{(l)}, \tag{5}$$

where

$$N_\gamma = \frac{4\pi\hbar c^2}{\Omega\omega^2} \left| \frac{\mathrm{Tr}\,\lambda_{ij}}{\dfrac{\partial \Lambda}{\partial \omega}} \right|, \tag{6}$$

λ_{ij} and Λ are the cofactors and determinant of the Maxwell operator Λ_{ij}

$$\Lambda_{ij} = \frac{c^2 K^2}{\omega^2} (K_i K_j - \delta_{ij}) + \varepsilon_{ij},$$

$$\mathbf{K} = \mathbf{k}/|\mathbf{k}|,$$

where ε_{ij} is the dielectric tensor of the medium. The quantum-mechanical form of the dielectric tensor $\varepsilon_{\alpha\beta}(\mathbf{k}, \omega)$ for a plasma in a uniform magnetic field has been worked out by Kelly (1964). However, since the use of the exact formulas would make the problem exceedingly complicated and since we want to compare our results with those of ARW, where the classical expressions for $\varepsilon_{\alpha\beta}$ have been used, we will choose here the $\varepsilon_{\alpha\beta}$ given by Stix (1962). In this way we neglect both the effects of the spin and of the spatial dispersion. The spin effects have been shown to be unimportant by Burt and Wahlquist (1962) and an evaluation of the importance of the terms depending on k^2 for $\varepsilon_{\alpha\beta}$ has been given by ARW, who conclude that they can be omitted in the region of astrophysical interest. Following Stix (1962) we have:

$$\begin{aligned}
&\varepsilon_{\alpha\beta} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}, \\
&S = \tfrac{1}{2}(R+L); \quad D = \tfrac{1}{2}(R-L) \\
&R = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega}{\omega-\omega_c}; \quad L = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega}{\omega+\omega_c} \\
&P = 1 - \omega_p^2/\omega^2 \\
&\omega_p^2 = 4\pi N e^2/m; \quad \omega_c = eH/mc
\end{aligned} \tag{7}$$

where N is the electron number density.

In this frame-work the refractive index $n = c|\mathbf{k}|/\omega$ satisfies the fourth-order equation

$$An^4 - Bn^2 + C = 0, \tag{8}$$

with

$$\begin{aligned}
A &= S\sin^2\theta + P\cos^2\theta, \\
B &= RL\sin^2\theta + SP(1+\cos^2\theta), \\
C &= RPL.
\end{aligned}$$

The solutions of Equation (8) are given by

$$\begin{aligned}
n^2 &= (B \pm F)/2A, \\
F^2 &= B^2 - 4AC.
\end{aligned} \tag{9}$$

Astrom (1950) and Allis *et al.* (1963) have suggested a more convenient form.

$$\tan^2\theta = \frac{-P(n^2-R)(n^2-L)}{(Sn^2-RL)(n^2-P)}, \tag{10}$$

which shows that at $\theta=0$, either $n_0^2=R$ (ordinary mode, O, $l=1$) or $n_x^2=L$ (extra-ordinary mode, X, $l=2$). Analogously at $\theta=\pi/2$, $n_0^2=P$ and $n_x^2=RL/S$. It can be easily shown that $\mathrm{Tr}\,\lambda_{ij}$ is given by

$$\mathrm{Tr}\,\lambda_{ij} \equiv \lambda_{ss} = n^4 - n^2(P+A+2S) + RL + 2SP. \tag{11}$$

The sum over l in Equation (5) only means sum over the two possible values of n^2. The general form of the polarization vector e_μ has been worked out by Melrose (1968) who gives the following form

$$e_i = \frac{\lambda_{ij}c_j}{\sqrt{c_i^*\lambda_{ij}c_j\lambda_{ss}}}.$$

As usual the index $l=1$, 2 is implicity contained in λ_{ij} through the refractive index n. The parameters c_j form a set of complex numbers, one of which needs to be non-zero. Choosing

$$\mathbf{c} = (0, i, 0),$$

we obtain after some algebra

$$\mathbf{e} = \frac{1}{\sqrt{1+\alpha^2}}\left\{\frac{D(P - n^2 \sin^2\theta)}{SP - An^2}, \quad i, \quad \frac{-Dn^2 \sin\theta\cos\theta}{SP - An^2}\right\},$$
$$\alpha = \frac{PD\cos\theta}{SP - An^2}. \tag{12}$$

Before ending this section we will give the values of $\mathbf{e}$ for $\theta=0$, and $\theta=\pi/2$, because of their later use.

From Equations (10) and (12) we obtain at $\theta=0$

$$\mathbf{e}(O) = \frac{1}{\sqrt{2}}\{1, \quad i, \quad 0\},$$
$$\mathbf{e}(X) = \frac{1}{\sqrt{2}}\{-1, \quad i, \quad 0\}; \tag{13}$$

and at $\theta=\pi/2$

$$\mathbf{e}(O) = \{0, \quad 0, \quad i\},$$
$$\mathbf{e}(X) = \{0, \quad 1, \quad 0\}. \tag{14}$$

4. Computation of the Energy Loss

We proceed now in the computation of the energy loss by means of Equation (3). This can be evaluated

$$S = \frac{g}{e\sqrt{2}}\frac{N_\gamma^{1/2}}{\Omega\hbar c}(2\pi)^4\delta^4(p_1/\hbar + p_2/\hbar - k) \times \sum_{l=1}^{2}\left[\Pi_{\alpha\beta}(\mathbf{k},\omega)\, e_\alpha\right]\bar{u}(p_2)\,\gamma_\beta(1+\gamma_5)\,v(p_1), \tag{15}$$

where the polarization tensor

$$\Pi_{\alpha\beta}(\mathbf{k}\omega) = \frac{e^2}{\hbar c}\frac{1}{(2\pi\hbar)^4}\int \mathrm{d}^4p\,\mathrm{Tr}\{\gamma_\alpha G(p)\,\gamma_\beta G(p+k)\}$$

contains all the information about the magnetic field, through its relation with the dielectric tensor (Tsytovich, 1961)

$$\Pi_{ij} = \frac{i}{4\pi}\left(\frac{\omega}{c}\right)^2[\varepsilon_{ij} - \delta_{ij}], \quad \Pi_{i4} = i\left(\frac{\omega}{c}\right)^{-1}\Pi_{ij}k_j,$$
$$\Pi_{44} = -\left(\frac{\omega}{c}\right)^{-2}\Pi_{ij}k_ik_j. \tag{16}$$

The use of Equations (16) makes it possible to avoid the direct computation of the polarization tensor $\Pi_{\alpha\beta}$, which requires the use of the exact Green function for an electron in a magnetic field, $G(p)$. Before proceeding we have, however, to comment on the absence of the electron axial part in the weak interaction Lagrangian, Equation (2). This part would have produced an expression quite similar to Equation (15) with $\Pi_{\alpha\beta}$ replaced by the corresponding $\Pi^5_{\alpha\beta}$. In this case however we cannot use anything similar to Equations (16) and we must compute $\Pi^5_{\alpha\beta}$ directly from the definition. Due to the very complicated expression for $G(p)$ this would be a rather formidable task. ARW have shown that in absence of a magnetic field the axial vector part gives a small contribution with respect to the vector one. We will assume here that the same is true even in the presence of a magnetic field. Squaring S, integrating over $\mathrm{d}^3p_1\,\mathrm{d}^3p_2$, the neutrino final momenta, and using the relation (Lenard, 1953)

$$c^2\iint \frac{\mathrm{d}^3p_1\,\mathrm{d}^3p_2}{2E_1 2E_2}\,p_1^\alpha p_2^\beta \delta^4(p_1+p_2-q)=\frac{\pi}{24}\left[\delta_{\alpha\beta}q^2+2q_\alpha q_\beta\right]$$

we get, after summing over the neutrino spins

$$\begin{aligned}\frac{\overline{|S|^2}}{\Omega T} &\equiv \sum_{\text{spin}}\frac{\Omega^2}{(2\pi\hbar)^6}\iint \mathrm{d}^3p_1\,\mathrm{d}^3p_2\,\frac{|S|^2}{\Omega T}\\ &= -\left(\frac{S_0}{\hbar\omega}\right)\sum_{l=1}^{2}\left|\frac{\lambda_{ss}}{\omega\dfrac{\partial\Lambda}{\partial\omega}}\right|\left[\Pi_{\alpha\beta}\Pi^*_{\varrho\sigma}e_\alpha e^*_\varrho\right]\left[\delta_{\mu\nu}k^2+2k_\mu k_\nu\right]\\ &\quad\times \mathrm{Tr}\left\{\gamma_\mu\gamma_\beta(1+\gamma_5)\gamma_\nu(1-\gamma_5)\gamma_4\gamma_\sigma\gamma_4\right\},\end{aligned}\tag{17}$$

with

$$S_0=\frac{1}{48}\frac{g^2c}{e^2}\frac{1}{\Omega}.$$

The energy loss is now easily found to be (per unit solid angle)

$$Q(\theta)=\frac{\Omega}{(2\pi)^3}\int|\mathbf{k}|^2\,\mathrm{d}|\mathbf{k}|\,\frac{\overline{|S|^2}}{\Omega T}\,\hbar\omega f(\omega)\quad\left(\frac{\text{erg}}{\text{cm}^3\text{-sec-sterad}}\right),$$

where

$$f(\omega)=\left\{\exp\left(\frac{\hbar\omega}{k_BT}\right)-1\right\}^{-1}.$$

Substituting Equation (17) we then obtain

$$\begin{aligned}Q(\theta)=\frac{-1}{12\pi^3\alpha}\left(\frac{g^2}{\hbar}\right)\int\frac{\mathrm{d}\omega}{\omega}\int|\mathbf{k}|^2\,\mathrm{d}|\mathbf{k}|\left(|\mathbf{k}|^2-\frac{\omega^2}{c^2}\right)\\ \times(-1)^{\delta_{\beta4}}\sum_{l=1}^{2}\left[\Pi_{\alpha\beta}\Pi^*_{\varrho\beta}e_\alpha e^*_\varrho\right]\left|\frac{\lambda_{ss}}{\dfrac{\partial\Lambda}{\partial\omega}}\right| f(\omega)\,\delta\left[\omega-\omega(\mathbf{k})\right],\end{aligned}\tag{18}$$

$$\alpha=e^2/\hbar c=1/137.$$

We have explicitly included the identity

$$\int\delta(x)\,\mathrm{d}x=1$$

to recall that ω is a function of $\mathbf{k}$ through the dispersion relation

$$|\mathbf{k}|^2 c^2 = \omega^2 n^2 .$$

The quantity

$$M = \frac{\delta[\omega - \omega(\mathbf{k})]}{(\partial \Lambda/\partial\omega)} = \frac{1}{|\partial\Lambda/\partial n^2|_{\Lambda=0}}\,\delta(n^2 - n_l^2)$$

can be simplified to

$$\Lambda \equiv \mathrm{Det}(\Lambda_{ij}) = An^4 - Bn^2 + C,$$

$$\partial\Lambda/\partial n^2 = 2An^2 - B = \pm F \quad \text{for} \quad \Lambda = 0,$$

$$M = 1/|F|\,\delta(n^2 - n_l^2).$$

Performing the $\mathbf{k}$-integration in (18), we finally obtain

$$Q(\theta) = \frac{1}{24\pi^3\alpha}\left(\frac{g^2}{\hbar c^5}\right)\sum_{l=1}^{2}\int_{\omega_0}^{\infty} \mathrm{d}\omega\,\omega^4 n_l(1 - n_l^2)\left|\frac{\lambda_{ss}}{F}\right| \times [(-1)^{\delta_{\beta 4}}\Pi_{\alpha\beta}\Pi^*_{\varrho\beta}e_\alpha e^*_\varrho]\,f(\omega). \tag{19}$$

The lower limit ω_0 in the integral is obtained by imposing the condition that n_l is real. In order to simplify the final expressions we will consider here only the cases of parallel ($\theta=0$) and perpendicular ($\theta=\pi/2$) propagations. For $\theta=0$, using Equations (13) and (16) we have

$$(-1)^{\delta_{\beta 4}}\Pi_{\alpha\beta}\Pi^*_{\varrho\beta}e_\alpha e^*_\varrho = \frac{1}{(4\pi)^2}\left(\frac{\omega_p}{c}\right)^4 \frac{\omega^2}{(\omega \mp \omega_c)^2}.$$

The upper sign corresponding to the extra-ordinary mode. On the other hand $|\lambda_{ss}/F|=1$ for both modes and therefore we have

$$Q(0) = \bar{Q}\omega_p^4 \sum_{l=1}^{2}\int_{\omega_0}^{\infty} \mathrm{d}\omega\,\frac{\omega^6}{(\omega \mp \omega_c)^2}\,n_l(1 - n_l^2)\,f(\omega), \tag{20}$$

where n_l is given by the expression

$$n_l^2 = 1 - \frac{\omega_p^2}{\omega^2}\frac{\omega}{\omega \mp \omega_c}$$

and

$$\bar{Q} = \frac{1}{12\alpha}\frac{1}{(2\pi)^5}\frac{g^2}{\hbar c^9}.$$

Putting $\omega_c=0$ in Equation (20) and integrating over the solid angle, we obtain the formula given by Zaidi (1965). For $\theta=\pi/2$ and for the ordinary mode we have

$$(-1)^{\delta_{\beta 4}}\Pi_{\alpha\beta}\Pi^*_{\varrho\beta}e_\alpha e^*_\varrho = \frac{1}{(4\pi)^2}\left(\frac{\omega_p}{c}\right)^4.$$

Since again $|\lambda_{ss}/F|=1$, we get

$$Q_0(\pi/2) = \bar{Q}\omega_p^4 \int_{\omega_p}^{\infty} \mathrm{d}\omega\, \omega^4 n_0 (1 - n_0^2) f(\omega), \tag{21}$$

where

$$n_0^2 = 1 - \omega_p^2/\omega^2 .$$

Thus for $\theta=\pi/2$ and for the ordinary mode the energy loss is independent of the magnetic field. Finally, for $\theta=\pi/2$ and for the extra-ordinary mode we get

$$Q_X(\pi/2) = \bar{Q}\omega_p^4 \int_{\omega_0}^{\omega} \mathrm{d}\omega \frac{\omega^8}{(\omega^2 - \omega_c^2)^2} n_X (1 - n_X^2) \left[(1 - n_X^2)\left(\frac{\omega_c}{\omega}\right)^2 + 1\right] f(\omega), \tag{22}$$

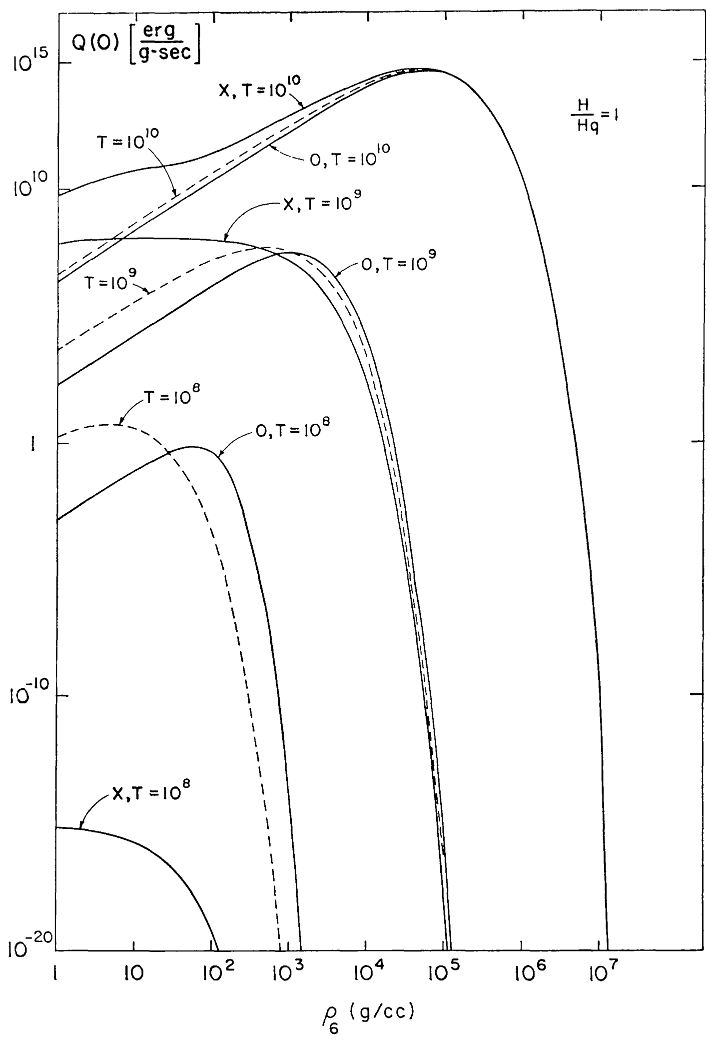

Fig. 2. Energy loss per unit solid angle vs. ϱ_6 and different temperatures for $H=H_q=4.41\times10^{13}$ G, and $\theta=0$. As explained in the text the symbols O and X stand for ordinary and extra-ordinary modes. The dashed lines correspond to $H=0$.

where

$$n_X^2 = 1 - \left(\frac{\omega_p}{\omega}\right)^2 \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_p^2 - \omega_c^2}.$$

5. Numerical Results and Conclusions

The expressions (20), (21) and (22) have been evaluated numerically for several values of the parameters $\varrho_6 \equiv 10^{-6}\ \varrho/\mu_e$, $\mu_e = Z/A$, H, T. In Figures 2 and 3 we have plotted our results for $H/H_q = 1$, $H_q = m^2c^3/e\,\hbar = 4.414 \times 10^{13}$ G and several temperatures. The quantity plotted is the energy loss per unit mass, which is obtained from Equations (20), (21) and (22), after dividing them by the density ϱ. The energy loss in the absence of the field is also shown for the sake of comparison.

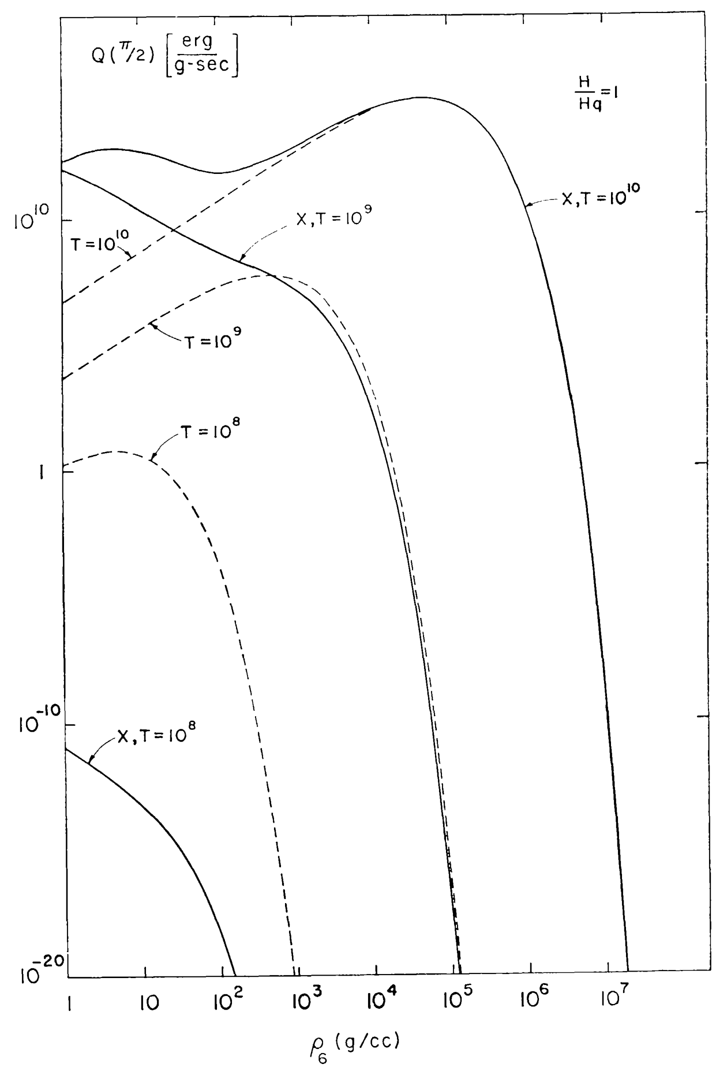

Fig. 3. The same as in Figure 2 for $\theta = \pi/2$. The ordinary mode coincides with the $H = 0$ case equation.

In general it can be seen that the effect of the magnetic field is relevant only in the low density region, and at extremely high fields. In fact, even for fields as high as 10^{-2} H_q, the energy losses are practically equal to those without magnetic field in the whole region $1 \leqslant \varrho_6 \leqslant 10^7$. This is due to the fact that the magnetic field always appears through the quantity ω_c, which in all the formulas is compared with quantities of the order or greater than ω_p. The ratio ω_c/ω_p is usually very small except in the regions we have indicated before. Actually, in the regions where fully relativistic formulas have to be used ω_c must be replaced by the corresponding relativistic quantity which turns out to be $\omega_c \varrho_6^{-1/3}$. This, of course, has again the effect of diminishing the size of the corrections to the zero field case in the region of high values of ϱ.

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